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**NEW YORK UNIVERSITY**
Courant Institute of Mathematical Sciences
Division of Electromagnetic Research

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# Far Field Amplitudes and Inverse Diffraction Theory

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## 1. SUMMARY

In electromagnetic and acoustic diffraction problems the behavior of the plane wave far field scattering amplitude is of crucial importance, both theoretically and practically. In the present paper we present some new properties of this function for plane wave excitation, in the case of two dimensional scattering and with special reference to the _inverse_ diffraction problem.

The function in question, which will be defined analytically below, is a continuous analogue of a scattering matrix. For a given scatterer and frequency, it depends on two variables, the angle of incidence $\theta_o$ of the plane wave, and the angle of observation $\theta$ of the far diffracted field at infinity. We call it $f(\theta,\theta_o)$. In the first part of the analysis, [section 3] we consider determinants formed from the values of this function. We deduce necessary and sufficient geometrical conditions (as to the geometry of the scatterer) for the vanishing of such determinants, under specific hypotheses as to the boundary conditions being fulfilled. These conditions show that for an acoustically _soft_ obstacle no third order determinant formed by using three different angles can vanish, and we conjecture the same result for all odd order determinants. Even order determinants can vanish.

In the second part of the analysis [section 4] we discuss the case where $f(\theta,\theta_o) = f(\theta - \theta_o)$ in the case of a soft obstacle. Such an amplitude arises when the scatterer is a circle, as is well known. We show that this is the _only_ possible cause. This is an explicitly

solvable problem in the exact Inverse Diffraction Theory.

The analysis of sections 3 and 4 is preceded [section 2] by a statement of definitions and known results, and the proof of a basic lemma.

## 2. DEFINITIONS AND PRELIMINARY LEMMAS

Let a plane wave $\phi(r,\theta;\theta_0) = \exp ikr\cos(\theta-\theta_0)$, traveling in the direction $\theta_0$, be incident on a scatterer S, contained within some circle of radius a, say, whose center is the origin of both polar and cartesian coordinates. Then the total field, produced by the interaction of the incident and diffracted fields, is denoted by $u(r,\theta;\theta_0)$, obeys the Helmholtz[1] equation $\Delta u + k^2 u = 0$, and can be written in the form

$$u(r,\theta;\theta_0) \equiv \phi(r,\theta;\theta_0) + v(r,\theta;\theta_0) \tag{1}$$

where $v(r,\theta;\theta_0)$ is the diffracted field, which must be regular outside S and be an outgoing wave function. We can write

$$u(r,\theta;\theta_0) = \exp ikr\cos(\theta-\theta_0) - \frac{i}{4}\oint \left[H_0(k\overline{PQ})\frac{\partial v}{\partial n_Q} - v(Q)\frac{\partial H_0}{\partial n_Q}(k\overline{PQ})\right] dS_Q, \tag{2}$$

where Q denotes a point on S, P an observation point, $\overline{PQ}$ their distance, $\partial/\partial n$ the outward normal derivative, and $H_0(kr)$ is the Hankel function of the first kind. (Here a time dependence $\exp(-i\omega t)$ has been assumed and then suppressed.) If we let $r \to \infty$ then we have the asymptotic formula

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[1] $k = \omega/c$ where $\omega$ is the frequency of the assumed monochromatic wave and c is the velocity of propagation.

$$v \to \tilde{H}_o(kr)\, f(\theta,\theta_o) ; \quad \tilde{H}_o \equiv \sqrt{\frac{2}{\pi kr}}\; e^{i(kr - \frac{\pi}{4})} , \tag{3}$$

and this defines the far field amplitude function $f(\theta,\theta_o)$. Comparison with (2) shows that

$$f(\theta,\theta_o) = -\frac{i}{4} \oint \left[ e^{-ikr_Q \cos(\theta-\theta_o)} \frac{\partial v}{\partial n}(Q;\theta_o) - v(Q;\theta_o) \frac{\partial}{\partial n} e^{-ikr_Q \cos(\theta-\theta_o)} \right] dS(Q).$$

(4)

The function $f(\theta,\theta_o)$ has the following well-known properties:

$$f(\theta,\theta_o) = f(\theta_o + \pi, \theta + \pi) , \tag{5}$$

$$\frac{1}{2\pi} \int_0^{2\pi} |f(\theta,\theta_o)|^2 d\theta = -\mathrm{Re}\, f(\theta,\theta_o) . \tag{6}$$

Equation (5) expresses reciprocity and may be obtained as a limiting form of the reciprocity theorem for Green's function. Equation (6) is the well-known forward amplitude theorem, often quoted or proved in the literature. The results stated so far are well known. A more general result will now be given in the form of a lemma, based on definitions which will be used in the rest of the paper.

Lemma: Let

$$u_i \equiv u(r,\Theta;\Theta_i) \qquad \phi_i \equiv \exp ik(x\cos\Theta_i + y\sin\Theta_i)$$

$$f_i \equiv f(\Theta,\Theta_i) \qquad u_i = \phi_i + v_i \tag{7a}$$

$$v_i \equiv v(\Theta,\Theta_i)$$

and let $\{A_i\}$ be n complex numbers, and let

$$U \equiv \sum_1^n A_i u_i \ , \quad V \equiv \sum A_i v_i \ , \quad \Phi \equiv \sum A_i \phi_i \ , \quad F \equiv \sum A_i f_i \ ,$$

$$f_{ij} \equiv f(\Theta_i,\Theta_j) \ , \quad F_i \equiv \sum A_j f_{ij} \tag{7b}$$

Then

$$\operatorname{Im} \oint_S V\frac{\partial V^*}{\partial n}dS = 2\sum_i \sum_j A_i^* A_j (f_{ij} + f_{ji}^*) \equiv -\frac{2}{\pi}\int_0^{2\pi} |F|^2 d\Theta \ , \tag{7c}$$

if it is assumed that on S we have, for all i, the boundary condition

$$d(S)u_i + \lambda(S)\frac{\partial u_i}{\partial n} = 0 \ ; \quad \operatorname{Im} \lambda(S)/d(S) = 0 \ , \tag{7d}$$

or, more generally, if the boundary conditions are such that

$$\operatorname{Im} \oint_S U\frac{\partial U^*}{\partial n}dS = 0 \quad \text{for arbitrary } \{A_i\} \ .$$

Proof: Under the given hypotheses we have $\operatorname{Im} \oint U \frac{\partial U^*}{\partial n} dS = 0$, while it is easily checked that $\operatorname{Im} \oint \frac{\partial \Phi^*}{\partial n} dS = 0$, since $\Phi$ is regular in the interior of S. Thus

$$0 = \operatorname{Im} \oint_S U \frac{\partial U^*}{\partial n} dS = \operatorname{Im} \left[ \oint \left( V \frac{\partial \Phi^*}{\partial n} + \Phi^* \frac{\partial V}{\partial n} \right) dS + \oint V \frac{\partial V^*}{\partial n} dS \right] . \qquad (7e)$$

We have,[2] by definition of $f_{ij}$, and use of (4)

$$f_{ij} = \frac{1}{4i} \oint \left( \frac{\phi_i^* \partial v_j}{\partial n} - \frac{\partial \phi_i^* v_j}{\partial n} \right) dS .$$

Therefore

$$4i \sum A_i^* F_i \equiv 4i \sum \sum A_i^* A_j f_{ij} = \oint \left( \Phi^* \frac{\partial V}{\partial n} - V \frac{\partial \Phi^*}{\partial n} \right) dS .$$

If we take the imaginary part of the last expression we find precisely the first half of (7c), in virtue of (7e). Now using the fact that for large r, $v_i \rightarrow \tilde{H}_o(kr) f_i$ and hence $V \rightarrow \tilde{H}_o(kr) F$, and using the fact that

$$\operatorname{Im} \oint_S V \frac{\partial V^*}{\partial n} dS = \operatorname{Im} \oint_{S_\infty} V \frac{\partial V^*}{\partial n} dS ,$$

where $S_\infty$ is a circle at ∞, we can easily verify that the second equation in (7c) is correct.

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[2] Note that i indicates an integer when used as a subscript, while otherwise i means $\sqrt{-1}$ .

Q.E.D.

3. Equation (4) shows that $f(\theta,\theta_o)$ is an analytic function of $\theta$. Thus it is uniquely determined by its values on any dense set. The problem of inverse diffraction is to determine S given $f(\theta,\theta_o)$, for a fixed k. It is of interest to consider the case when $\theta_o$ is fixed or also, the case of variable $\theta,\theta_o$. The program to be followed would be to use f to determine v and hence u, and then to find the locus on which u obeys the boundary conditions. This is obviously difficult in general. In the present paper we prove some simple results which relate f and S under special circumstances. The present section is concerned with properties which arise from a consideration of determinants formed from $f(\theta,\theta_o)$, i.e. $\det(f_{ij})$. We treat the boundary conditions $u = 0$, $\partial u/\partial n = 0$ in this paper. The main results will now be listed.

Case a). Soft obstacles, boundary condition $u_i = 0$. We prove the following theorems.

Theorem Ia. If $\det(f_{ij}) = 0$, where $\{\theta_i\} = \theta_1,\theta_2,\ldots,\theta_n$ are n different angles modulo $2\pi$, and if no subdeterminant vanishes, then there exist n nonvanishing constants $\{A_m\}$ such that the necessary and sufficient condition for (x,y) to be on S is that

$$\sum_{1}^{n} A_m \exp ik(x \cos \theta_m + y \sin \theta_m) = 0 . \tag{8a}$$

The $A_m$ are subdeterminants of $\det(f_{ij})$. If S contains a closed curve then the wave number k is an eigenvalue for the interior of that curve. Equation (8a) can be interpreted to mean that S lies on an arc of an interference pattern.

Theorem IIa. No third order determinant of the above kind can vanish, since (8a) is impossible on a real arc for $n = 3$. The latter assertion, to be proved below, means that three plane waves can't interfere on an arc.

Theorem IIIa. For any integer n such that there is no real arc along which (8a) holds, $\det(f_{ij})$ cannot vanish under the circumstances described in the hypothesis of Theorem Ia.

Theorem IVa. If n is even it is always possible to find a real S on which (8a) is fulfilled. (For example (8a) is fulfilled along any segment of any node of any eigenfunction of a square membrane for which k is an eigenvalue, as is easily seen. It is well known that some of these nodes are curved. See [1], p. 314 for examples.)

Conjecture. If n is odd it is impossible to find a real arc along which (8a) is satisfied.

If the above conjecture can be proved, then Theorem IIIa implies that no odd order determinant of $f_{ij}$ can ever vanish, if no subdeterminants vanish.

Case b). Hard obstacle, boundary condition $\partial u_i/\partial n = 0$. We have the following results.

Theorem Ib. Theorem Ia is valid if we replace (8a) by

$$\sum_m A_m \frac{\partial}{\partial n} [\exp ik(x \cos \theta_m + y \sin \theta_m)] = 0 . \qquad (8b)$$

On the other hand, Theorem IIa is not valid when this substitution is made, although Theorem IIIa is. This follows from the fact that a sum of three plane waves can be easily constructed so as to satisfy (8b) on any segment of the x axis. Probably the conjecture stated above is incorrect for a hard obstacle.

Case c). Reactive or capacitive obstacle. The boundary condition is $d(S)\frac{\partial u}{\partial n} + \lambda(S)u = 0$ with $d(S)/\lambda(S)$ real. One can derive a differential equation for the obstacle but the discussion is beyond the scope of the present paper.

Some additional results arise in the course of the proofs of the above assertions. We find, under the hypotheses of Ia or Ib,

$$\sum A_m f(\theta,\theta_m) = 0 \qquad \text{for all } \theta , \qquad (9)$$

$$\sum A_m V(r,\theta;\theta_m) = 0 \qquad \text{for all } \theta , \qquad (10)$$

$$\sum A_m f(\theta_m + \pi, \theta + \pi) = 0 \qquad \text{for all } \theta . \qquad (11)$$

The last equation follows from reciprocity and it expresses a forced relation between the scattered field in certain characteristic directions when the direction of excitation, $\theta + \pi$, is arbitrary. The well known evenness or oddness of the scattered far field produced by a conducting strip is an example if (11) for n = 2. Other[3] geometries possessing this property for the case of u = 0, are any combination of parallel, vertical, zero thickness conductors of any size, placed any number of half wave lengths apart, when $\theta_1 = 0$, $\theta_2 = \pi$. In the case $\partial u/\partial n = 0$, these segments can be closed by horizontal segments placed arbitrarily. Thus we obtain the case of a rectangle, for example. Another case of interest here is the case of a finite or infinite grating of rectangular grooves of depth = 1/2 wave length, cut out of the conducting half space $x \geq 0$; provided we are in Case b), equation (10) shows that the incident wave plus the usual reflected wave, that is, $e^{ikx} + e^{-ikx}$, satisfies all conditions. There is no diffracted field. This is easily verified a posteriori.

It should be noted that equations (9) and (10) are deduced from the determinantal condition and lead to our main results. But, of course, (9) and (10) also follow from the main results. In fact, if (8a) or (8b) are satisfied, then application of the boundary conditions to

$$\sum A_m V(r,\theta;\theta_m) \equiv F$$

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[3] See ref [2] where some of these results are derived for the case of n = 2.

proves (10) if the uniqueness theorem is applied to the radiating wave function F. Equation (9) is the asymptotic form of equation (10).

We now proceed to prove the results already stated. Consider the function V defined in equation (7b). It is an outgoing wave function regular in the exterior of the obstacle S, and by (7d) we have, on S

$$d(S) \sum A_i u_i + \lambda(S) \sum A_i \frac{\partial u_i}{\partial n} = 0 . \qquad (12)$$

Now, by the definitions in (7b) U = Φ + V. Consequently, on S,

$$d(S)\Phi + \lambda(S) \frac{\partial \Phi}{\partial n} = - \left\{ d(S)V + \lambda(S) \frac{\partial V}{\partial n} \right\} . \qquad (13)$$

[For a hard obstacle $d(S) = 0$. For a soft obstacle $\lambda(S) = 0$.] Now suppose $\det(f_{ij}) = 0$, and no subdeterminant vanishes. Then we set

$$A_j = \text{cofactor of } f_{1j} \neq 0 , \qquad (14)$$

and we easily deduce that the double sum in (7c) vanishes. Hence the function F vanishes. This is equation (9). Since F is the leading term in the expansion of V for large r, we deduce from Rellich's Lemma that V vanishes identically for this choice of $\{A_j\}$, which proves equation (10). This implies that the right-hand side of (13) vanishes identically. If the left-hand side of (13) is specialized to the case of soft (hard) obstacles, Theorem Ia (Ib) follows (cf. (8a) and (8b)). Notice that when condition (7d) is used and not specialized, then we get a differential equation for S from the vanishing of the left side of (13), using $\partial y/\partial n = dx/dS$. The converses of Theorems Ia and Ib follow from the fact that if (8a) or (8b) is assumed, and none of the $A_i$ is zero, then the left-hand side of (13) vanishes. Hence so does the right side. Hence

$$\mathrm{Im} \oint V \frac{\partial V}{\partial n} dS = 0 .$$

Hence $F \equiv 0$. Hence, using

$$F = \sum A_i f(\theta, \theta_i) ,$$

we deduce equation (9), of which equation (10) is an easy consequence. If we set $\theta = \theta_j$ in (9), we have n homogeneous equations for $j=1,\ldots,n$. Since none of the $A_i$ vanish it follows that $\det(f_{ij}) = 0$, and no subdeterminant is zero. As to equation (11), it follows from (9) by reciprocity.

We proceed to Theorem IIa. Let S be a real arc along which (8a) holds.

Let $\gamma_m(S) \equiv \frac{dx}{dS} \cos \theta_m + \frac{dy}{ds} \sin \theta_m$, and let $\dot{\gamma}_m \equiv d\gamma_m/dS$ .

Construct two additional linear equations for the $A_m$ by differentiating

(8a) twice with respect to arc length. Then it is readily verified that the determinant of the coefficients ia a multiple of

$$\begin{vmatrix} 1 & 1 & 1 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_1^2 & \gamma_2^2 & \gamma_3^2 \end{vmatrix} + \sqrt{-1} \begin{vmatrix} 1 & 1 & 1 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} .$$

The first term is a Vandermonde determinant. Its vanishing shows that two of the $\gamma$'s must be equal. Since this must be true on a continuum of S, it follows that the corresponding pair of angles are equal modulo $2\pi$, contrary to hypothesis. Hence, there is no real arc on which (8a) is satisfied nontrivially for $n = 3$. But if $\det(f_{ij}) = 0$ for $n = 3$, for the case of a real obstacle, then (8a) must be satisfied. This contradiction proves IIa. Unfortunately, this seems too complicated a method for higher values of n.

Theorem IIIa is now obvious, since it is the negation of Theorem Ia.

Theorem IVa has been explained adequately, if briefly, and so have the conjecture and the related remarks in re case b).

## 4. THE CIRCLE THEOREM:

$$f(\theta,\theta_o) = f(\theta - \theta_o).$$

The purpose of the present section is to prove the following theorem: If the far field amplitude $f(\theta,\theta_o)$ has the form $f(\theta - \theta_o)$, and if the boundary value problem giving rise to it is to find a function $v(r,\Theta;\Theta_o)$ such that

$$v \equiv e^{ikr\cos(\Theta-\Theta_o)} + v(r,\Theta;\Theta_o) = 0 \qquad (15)$$

on the scatterer S, and if

$$\underset{r\to\infty}{v} \to \tilde{H}_o^{(1)}(kr)f(\Theta-\Theta_o) \,, \qquad (16)$$

then S is a circle.

Proof: Let the coordinates be chosen so that the smallest circle, say C, which encloses the scatterer has the equation $r = a$. Then v is regular outside $r = a$, and we can write v in the form

$$v = \sum_{-\infty}^{\infty} v_n(\Theta_o)\frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} e^{in\Theta} \,, \qquad r \geqq a \,, \qquad (17)$$

where

$$v(a,\Theta;\Theta_o) = \sum_{-\infty}^{\infty} v_n(\Theta_o)e^{in\Theta}$$

and the function v is sufficiently regular so that for large r

$$v \to \tilde{H}_o(kr)\sum_{-\infty}^{\infty} \frac{v_n(\Theta_o)}{H_n^{(1)}(ka)} e^{\frac{in\pi}{2}} e^{in\Theta} \qquad (18)$$

Comparison with (16) shows that $v_n(\Theta_o)$ has the form $c_n\exp(-in\Theta_o)$.

Therefore we have

$$v(r,\theta;\theta_o) = \sum_{-\infty}^{\infty} c_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} e^{in(\theta-\theta_o)} \tag{19}$$

Hence, using the polar coordinate expansion for a plane wave we have

$$u(r,\theta;\theta_o) = \sum_{-\infty}^{\infty} i^n J_n(kr) e^{in(\theta-\theta_o)} + \sum_{-\infty}^{\infty} c_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} e^{in(\theta-\theta_o)}$$

or

$$u(r,\theta;\theta_o) = \sum_{-\infty}^{\infty} \left[ i^n J_n(kr) + c_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} \right] e^{in(\theta-\theta_o)} \tag{20}$$

Now, let $r \to a$. The circle $r = a$ touches the body S at some point at which $\theta = \beta$, say. Hence

$$u(a,\beta;\theta_o) = \sum_{-\infty}^{\infty} \left\{ [i^n J_n(ka) + c_n] e^{in\beta} \right\} e^{-in\theta_o} = 0. \tag{21}$$

Although $\theta$ is fixed at the value $\beta$, equation (21) holds for all $\theta_o$; hence it follows that

$$c_n = -i^n J_n(ka) . \tag{22}$$

But this implies that u vanishes for $r = a$ and all $\theta$. Therefore a solution is the circle $r = a$. Now, we have seen that

$$u \equiv \sum_{-\infty}^{\infty} i^n \left[ J_n(kr) - \frac{J_n(ka)}{H_n^{(1)}(ka)} H_n^{(1)}(kr) \right] e^{in(\theta-\theta_o)} \qquad (23)$$

vanishes everywhere on the circle $r = a$. However, the same proof also indicates that if $u$ vanishes at any other point, at which $r = b \neq a$, then $u$ vanishes on the entire circle $r = b$. We shall next prove that the function $u$ as given by (23) cannot vanish on two concentric circles. This will prove that all points of $S$ lie on the circle $r = a$.

To prove that if $u(a,\theta-\theta_o) \equiv 0$ then $u(b,\theta-\theta_o) \not\equiv 0$ unless $a = b$, we define

$$Z_n(kr) \equiv \operatorname{Re} \left[ J_n(kr) - \frac{J_n(ka)}{H_n(ka)} H_n(kr) \right] . \qquad (24)$$

Then, as remarked in the previous paragraph, $u(a,\theta-\theta_o) \equiv 0$ implies $Z_n(ka) = 0$, and $u(b,\theta-\theta_o) \equiv 0$ implies $Z_n(kb) = 0$. These relations hold for all values of $n$. Now define

$$F_n(kr,\theta) = Z_n(kr) \sin n\theta . \qquad (25)$$

Then $F_n(r,\theta)$ vanishes on the "rectangle" $B_n\left[r = a,\ r = b,\ \theta = 0,\ \theta = \frac{\pi}{r}\right]$. Hence $k$ is an eigenvalue for $B_n$. Now recall that $k$ is fixed. Recall also that the lowest eigenvalue of a membrane can be made arbitrarily large by decreasing the area. But the area of the rectangle $B_n$ tends to zero with increasing n. Hence $F_n$ cannot vanish on $B_n$ for all values of n. Since sin $n\Theta$ does vanish on $\Theta = 0$, $\Theta = \frac{\pi}{n}$, it follows that if $Z_n(ka) = 0$ for all n, $Z_n(kb) \neq 0$ for all n, unless $b = a$. This result of course implies that $u(r,\Theta-\Theta_o)$ cannot vanish on two different circles, as was to be shown.

We have proved that all points of the conductor S lie on a single circle $r = a$. To complete the proof that the conductor is a circle we have to show that all points of the circle $r = a$ are covered by the conductor, i.e. show that S is closed.

Assume, to the contrary, that there is a gap on the circle $r = a$. Observe that the known estimates of Bessel and Hankel functions for large order and fixed argument show that the series

$$\sum_{-\infty}^{\infty} i^n \left[ J_n(kr) - \frac{J_n(ka)}{H_n^{(1)}(ka)} H_n^{(1)}(kr) \right] e^{in(\Theta-\Theta_o)}$$

converges uniformly and absolutely for all r other than $r = 0$, and defines a regular wave function. This shows that the series provides the regular continuation of u into the region $r < a$. The existence of a gap in the conductor S therefore allows u to be continued into the interior of the circle and inexorably leads to a singularity at the origin where there is no conductor. This only can be avoided by excluding the origin, i.e. by the absence of a gap in the conductor. Hence the conductor S which can only lie on the circle r = a, is closed, and this completes the proof.

It should be noted that this section presents an example of arguments proving the uniqueness of the inverse problem of diffraction in a particular case. The general case deserves study.

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